APPENDIX C

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In 1905 Einstein envisaged energy of mass m as being equivalent to mass × velocity of light squared. According to the literature (Pais (1982)) Einstein's proof is as follows: "If a body gives off the energy L in radiation, its mass diminishes by L/c^2." This links energy with the velocity of light or the propagation of electromagnetic waves, and the conclusion drawn was that energy of mass must be related to the speed of light so that $L = mc^2$. Today, nearly 100 years later, the equation has seen no change except that the symbol for energy L is usually replaced by E. There has never been any doubt about the accuracy of the equation as long as the speed of light remains constant. However, recent advances in space science and satellite technology suggest that there are changes in both the speed of light and the rate of time, which poses the question: is energy of mass really proportional to the speed of light squared and how can we test the validity of Einstein's proof? This essay provides such a test, and the result is quite contradictory since it will show that energy of mass in Einstein's equation is not proportional to the velocity of light squared, but is proportional to a universal gravitational potential or gravitational tension ϕ_{univ}. It so happens that ϕ_{univ} at our reference position in space is identically equal to c^2 or $\phi_{univ} \equiv c^2$.

The Problem with $E=mc^2$

Einstein's equation $E = mc^2$ suggests that energy stored in mass is proportional to the velocity of light squared. In fact, the opposite is true. Energy stored in mass is **inversely** proportional to the velocity of light squared. For example, if energy of mass is proportional to the velocity of light squared, then energy of mass will be less in situations where velocity of light is slower, *e.g.* at the surface of the Sun or near a massive black hole where light is believed to slow down to zero preventing it from escaping the black hole's strong gravitational field.

Let us first examine the origin and meaning of $E = mc^2$ and what c^2 in Einstein's equation really stands for. It is important to know that c^2 in Einstein's equation equals energy per unit mass or $c^2 = E/m$ and that c^2 originates from the well known equation

$$\phi_{univ} = \frac{GM_{univ}}{R_{univ}} = c^2 \text{ (energy per mass)} \tag{146}$$

where ϕ_{univ} is the cosmic gravitational potential or gravitational tension of our Universe and R_{univ} is the radius of curvature of the Universe. M_{univ} is the total mass of matter within R_{univ} and G is the universal gravitational constant.

It is obvious that when the gravitational potential or tension ϕ_{univ} increases then c^2 also increases. Consequently, if c^2 refers to the speed of light the speed of light would also increase.

This is not so because when light encounters an increase in gravitational tension it slows down as is evident from light streaking near the increased gravitational field $\Delta\phi$ at the surface of the Sun. In fact the velocity of light near a gravitational mass M at a distance R is

$$v_{light} = c\left(\frac{\phi_{univ}}{\phi_{univ} + \Delta\phi}\right)^2 \tag{147}$$

where $\Delta\phi = GM/R$ and c the free space value of the speed of light.

This is an example of how mathematics sometimes can give a correct numerical answer but a misleading physical picture. The problem is that c^2 in Einstein's $E = mc^2$ has always been associated with the speed of light, when in reality c^2 in Einstein's equation has a different meaning. Although the numerical value and the physical dimensions of c^2 are the same as that of the speed of light squared, its interpretation is quite different. For example, length is not the same as surface area which is length squared. Another example is the amount of energy per unit mass required to lift a mass to a certain height h above the Earth's surface under the influence of the Earth's gravitational acceleration g

$$E/m = gh \quad (\text{meter/second}^2), \qquad (l^2/t^2) \qquad (148)$$

which means energy per mass but has the same dimensions as that of velocity squared, even though the physical process described does not involve velocity at all. In reality, c^2 in Einstein's equation and meter per second squared in the above equation represent gravitational potential or gravitational tension ϕ. Einstein's energy equation should therefore be written as

$$E = m\phi_{univ}, \quad \text{where} \quad \phi_{univ} = GM_{univ}/R_{univ} \equiv c^2. \qquad (149)$$

Here ϕ_{univ} is the cosmic gravitational tension, or the amount of Energy per unit mass E/m generated by the Universe. Note that ϕ_{univ} includes both the Earth's and the Sun's gravitational tension at our location here on Earth.

The Sun's gravitational tension at Earth is 100 million times weaker than ϕ_{univ} and the Earth's own gravitational tension at its surface is an additional 14 times less. The cosmic gravitational tension ϕ_{univ} determines the rate of time and it also establishes the speed of light and therefore acts as a propagation medium for electromagnetic waves. This means that any change in the cosmic gravitational tension also changes the speed of light.

The ratio of change in time and rate of clocks and physical processes is

$$\frac{\phi_{univ}}{\phi_{univ} + \Delta\phi} \quad \text{and} \quad \frac{\phi_{univ}}{\phi_{univ} - \nabla\phi}, \qquad (150)$$

where $\Delta\phi$ represents an increase in gravitational tension and $\nabla\phi$ a decrease in gravitational tension relative to an observer. For example, when a clock advances one second on Earth the same clock on the Sun's surface would read

$$t_{sun} = t_{earth}\frac{\phi_{univ}}{\phi_{univ} + \Delta\phi} = 0.99999788\ sec, \qquad (151)$$

where $\Delta\phi = Gm_{sun} / r_{sun}$ is the gravitational tension of the Sun added to ϕ_{univ} and neglecting the influence of the Earth's gravitational tension.

However, one second on the Sun will equal

$$t_{earth} = t_{sun} \frac{\phi_{univ} + \Delta\phi}{\phi_{univ}} = 1.00000211 sec, \qquad (152)$$

on Earth.

The slowdown of solar time has been measured by Snider (1972).

A reduction in gravitational tension, on the other hand, will speed up clocks or physical processes and the propagation of light. A clock raised to any height above the Earth's would therefore run faster because of the decrease in the Earth's gravitational tension $\nabla\phi$ with altitude. The rate of a clock at height h above ground relative to a clock on the Earth's surface is

$$t_{height} = t_{earth} \frac{\phi_{univ}}{\phi_{univ} - \nabla\phi} \qquad (153)$$

which has been verified by many experiments (Pound and Rebka (1960)) including the Mössbauer effect involving very sensitive measurements of atomic frequency spectra.

As a contrast to change in time the change in the propagation of light caused by variation in the cosmic gravitational tension is twofold. First, light is subject to the same slowdown in time as observed for clocks and physical processes described above. Second, it is also subject to the change in tension of the propagating medium. The result is that the speed of light is affected twofold, *i.e.*

$$v_{light} = c\left(\frac{\phi_{univ}}{\phi_{univ} + \Delta\phi}\right)^2 \text{ and } v_{light} = c\left(\frac{\phi_{univ}}{\phi_{univ} - \nabla\phi}\right)^2. \qquad (154)$$

These equations are in agreement with experimental data. One such experiment involved the timing of radar waves bouncing off Venus (Shapiro (1971)) while crossing the gravitational field of the Sun. The change in velocity and time by gravitational fields is often referred to as gravitational red shifts when generated by $\Delta\phi$ and blue shifts when produced by $\nabla\phi$. The velocity of light remains constant whenever the cosmic gravitational tension ϕ_{univ} is constant.

Since gravitational tension ϕ represents energy per unit mass, there are other processes that can increase or decrease the energy of mass, *e.g.* kinetic energy. A fast-moving jet aircraft will generate a level of energy per unit mass of υ^2, where υ is velocity of the jet. This will raise the gravitational tension of the jet aircraft by $\frac{1}{2}\upsilon^2$, (non-relativistic) thus slowing clocks and physical processes accordingly, so that the clock rate onboard the jet would be

$$t_{jet} = t_{earth}\frac{\phi_{univ}}{\phi_{univ} + \frac{1}{2}\upsilon^2}\,. \quad \text{(non-relativistic)} \qquad (155)$$

This is true only if the change in gravitational tension due to the jet's altitude is not considered. Experiments involving clocks on board jet aircraft (Hafele (1972)) verify Equation (155). Also, an increase in the half-life of decaying cosmic particles has been observed, which has been attributed to the high velocities of the particles as they enter the Earth's atmosphere. The time change due to velocity is known as time dilation.

Orbiting satellites are subject to both time dilation and gravitational blue shifts, which compete against each other, as shown in the following equation

$$t_{sat} = t_{earth}\frac{\phi_{univ}}{\phi_{univ} - \nabla\phi + \frac{1}{2}\upsilon^2}\,. \quad \text{(non-relativistic)} \qquad (156)$$

Both $\nabla\phi$ and υ^2 can vary due to the position of the satellite relative to the Sun's, Moon's and Earth's gravitational tensions. The orbital velocity υ of a satellite is determined relative to the fixed stars. Clocks on a GPS satellite (Ashby (2003)) orbiting at an altitude of 26500 km will run faster by about 4.92×10^{-10} second per second and clocks onboard the space station, which orbits at the much lower altitude of 380 km above Earth, will run slower by 2.88×10^{-10} second per second compared with clocks on the Earth's surface, ignoring the effect on clocks on Earth caused by the Earth's own rotational velocity. For relativistic velocities we need to replace the Newtonian $\frac{1}{2}\upsilon^2$ with

$$\phi_{rel} = \phi_{univ}\sqrt{\frac{\phi_{univ}}{\phi_{univ} - \frac{1}{2}\upsilon^2}} - \phi_{univ} \qquad (157)$$

Another interesting consequence of Equation (155) is Einstein's twin paradox, where a twin traveling in a space ship will age more slowly than the twin remaining on Earth, so that on the return from the voyage the returning twin will be younger than the one on Earth. Here $\nabla\phi$, the speedup of time, in Equation (155) can be replaced by the square of the escape velocity $(\upsilon_{esc}^2 = Gm_{earth}/r_{earth})$ required for the space ship to leave the Earth's gravitational field and υ^2 equals the square of the velocity of the space ship when traveling through space.

Conclusions

Replacing velocity of light squared in Einstein's equation $E = mc^2$ with ϕ_{univ} has several implications, two of which are mentioned below.

The slow-down of light in strong gravitational fields according to Equation (147) offers a simple explanation for the bending of light near gravitating bodies such as the Sun. The index of refraction n according to Snell's law or René Decartes' law is

$$n = \frac{c}{\upsilon} = \frac{\sin\alpha'}{\sin\alpha''} \text{ and } \frac{\Delta\upsilon}{c} = \frac{\Delta(\sin\alpha)}{\sin\alpha'} \tag{158}$$

where c and υ represent the speed of the incoming and retarded light respectively and α' and α'' are the angles of incidence and refraction respectively. Since the mean incident angle of light penetrating the Sun's gravitational equal-potentials is $45°$ then the total angle of refraction becomes

$$\alpha_{bend} = 2\left\{\alpha' - \sin^{-1}\left[\sin\alpha' - \Delta(\sin\alpha)\right]\right\} = 1.75053023 \text{ arc second} \tag{159}$$

for light grazing the Sun's surface. The factor "2" is necessary since light has to pass through two refractive indices, one at the entrance to and one at the exit from the gravitational field.

The outcome of Equation (147) also casts serious doubt on the existence of black holes, since the equation shows that light cannot slow to zero in order to be prevented from leaving a black hole's gravitational field no matter how strong the gravitational field.