

## THE HYDROGEN ATOM

The Hydrogen atom with its naturally fixed orbits has served as a testing platform for many atomic models but a flawless theory devoid of fuzzi factors that offers exact solutions is still lacking. Most quantum mechanical theories seem to employ momentum in their equations while in fact Planck's constant is not exactly compatible with momentum since it is an energy constant. Another problem is that Einstein's relativity theory falls short dealing with velocities created by loss of rest mass energy such as experienced by electrons falling into atomic orbits. By employing Planck's constant as an energy constant and by modifying Einstein's energy velocity equation to include velocities generated by loss of rest mass energy will put the orbits in their right places to a precision limited only by the accuracy of the physical constants.

Atoms have been thought of as miniature solar systems ever since the beginning of the 20th century. It was Ernest Rutherford (1911) who proved this point of view to be true through scattering experiments but it remained a mystery why the atomic orbits could only exist in certain well defined radii from the central nucleus. Niels Bohr, a student of Rutherford's, discovered that the well defined orbits in Hydrogen seemed to appear in certain steps of the angular momentum  $h = m_e v r$  where  $h$  is Planck's constant,  $m_e$  the electron's mass,  $v$  the orbital velocity and  $r$  the radius of the orbit. Soon thereafter, a remarkable discovery was made by Lois de Broglie, which would revolutionize not only atomic physic but particle physics as well. Lois de Broglie showed that moving particles, such as electrons, have wave properties similar to that of light rays and that their wavelengths, just like light, could be determined by Planck's constant and from the particles momentum or  $\lambda = h / (mv)$ . de Broglie also proposed that orbits can only exist at atomic radii where the orbital circumference  $2\pi r$  equals the electron's particle wavelength in wave lengths of  $\frac{1}{2}\lambda, 1\lambda, 1\frac{1}{2}\lambda, 2\lambda, 2\frac{1}{2}\lambda, 3\lambda$  etc. This motivated the Austrian physicist Erwin Schrödinger and others to formulate orbital equations based on wave properties and statistics which are about where we are today. Even though atomic physics has advanced significantly since Rutherford's, Bohr's and de Broglie's time together with Einstein's relativity, the equations do not fit measurements exactly. The problem is twofold. The first problem is

that Planck's constant is an energy constant and is not compatible with momentum so de Broglie's momentum equation  $\lambda = h/(mv)$  has to be replaced by the energy relation  $\lambda = hv/E$ . This becomes clear if we compare

$$\lambda_{energy} = \frac{v}{f} = \frac{hv}{E} \neq \lambda_{momentum} = \frac{\frac{1}{2}hv}{\frac{1}{2}mv^2} = \frac{h}{mv}.$$

Where  $f = E/h$  is the frequency of the particle associated with its energy? We know that Newton's energy relation  $\frac{1}{2}mv^2$ , from which the momentum is deduced, is non relativistic and not accurate at high velocities and therefore, any quantum theory based on particle momentum is doomed and cannot yield correct answers. The second problem is that we can not use Einstein's relativistic velocity equations because orbits in atoms are created by loss of rest mass energy whereas Einstein's relativity is only accurate for velocities generated by a gain in rest mass energy, see <http://aflb.ensmp.fr/AFLB-361/aflb361m657.pdf>. The purpose of this essay is to show that by applying relativistic velocities generated by loss of energy, together with de Broglie's wave theory based on energy rather than momentum, one is able to obtain accurate energies for the different orbits and wave lengths of  $n$  in the Hydrogen atom from

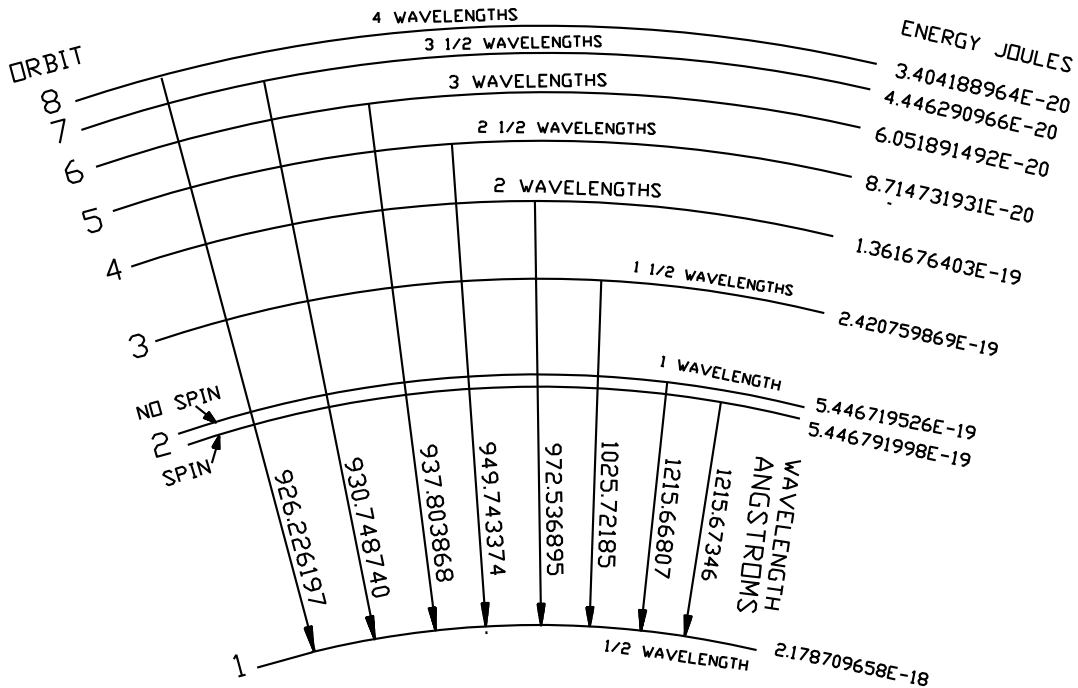
$$E_{orbit} = E_0 \left[ 1 - \sqrt{1 - \left( \frac{Zq^2}{n4\varepsilon_0 hc} \right)^2} \right] \times \frac{m_n}{(m_n + m_e)}. \quad (\text{Joules})$$

Orbital energies are listed and shown in the table and diagram below. The term  $m_n/(m_n + m_e)$  where  $m_n$  and  $m_e$  are the masses of the proton and electron respectively and reduces the orbital energy to that of the electron only.  $Z=1$  for Hydrogen,  $\varepsilon_0$  is the permittivity constant,  $q$  the electronic charge and  $c$  the speed of light.  $E_0$  is the electron's rest mass energy from  $E_0 = m_e c^2$ . Note that the equation treats the orbiting

electron both as a particle and wave at the same time which violates the Copenhagen Interpretation. offering exact solutions and

Orbit	$n = \text{Wave length}$	Energy Joules	eV
1	$\frac{1}{2}$	$2.178709658 \times 10^{-18}$	13.598440
2	1	$5.446719526 \times 10^{-19}$	3.399573
3	$1\frac{1}{2}$	$2.420759869 \times 10^{-19}$	1.510918
4	2	$1.361676403 \times 10^{-19}$	0.8498911
5	$2\frac{1}{2}$	$8.714731931 \times 10^{-20}$	0.5439305
6	3	$6.051891492 \times 10^{-20}$	0.3777291
7	$3\frac{1}{2}$	$4.446290966 \times 10^{-20}$	0.2775155
8	4	$3.404188964 \times 10^{-20}$	0.2124726

The first eight orbits in Hydrogen and their energies.



Energy diagram of the Hydrogen atom

An electron can fall from any orbit down to the ground state or orbit number 1, as shown by the arrows in diagram. The orbiting electron can also fall between different orbits which is not shown here. As the

electron falls in to a closer orbit, potential energy is lost to radiation. The amount of energy escaped, in the form of radiation equals the energy difference between the orbits. In the diagram the calculated wavelengths of the radiation liberated are listed next to each arrow. The spectrum of wavelengths is that of the Lyman series.

One interesting phenomenon is the electron's spin or rotation around its own axis. The spin will shift the orbital energies causing each orbit to split up into small fractions. This energy fragmentation can be seen in the emitted wavelengths and is called finestructure splitting. The effect of the electron's spin is shown in the diagram by the double line in orbit number 2 and can be explained by tidal forces similar to that of the Earth's tidal influence on the Moon. For example, consider the Moon being captured in its 28 day orbit around the Earth. At first the Moon would not rotate around its own axis and would show the same face relative to the fixed stars but it would appear to rotate once every 28 days relative to Earth. However, tidal forces between Earth and the Moon will soon bring this relative rotation to a stop so that the Moon now will show the same face to the Earth at all times but rotate once every 28 day around its axis relative to the fixed stars. The energy consumed in generating the Moon's rotation or spin around its axis relative to the fixed stars will cause the Moon's orbit to fall slightly closer to the Earth. An electron in orbit around the atomic nucleus will suffer the same fate and will eventually rotate with the same frequency around its axis as around its orbit. The energy ratio between the two rotations will therefore equal the ratio of the orbital radius to that of the electron's radius  $R/r_e$  where  $R$  is the distance between the electron and the nucleus. The maximum loss of energy due to the electron's spin in orbit 2, for example, in Hydrogen is therefore

$$E_{spin} = \frac{nr_e}{R} E = \frac{n8\pi\epsilon_0 E_{orb}^2 r_e}{q^2} \times \left[ \frac{m_n + m_e}{m_n} \right]^2 = 7.2550833 \times 10^{-24} \text{ Joules}$$

where  $E_{orb}$  is the electron's energy in orbit 2 and  $n=1$  is the wave number or wave length of orbit 2.  $E$  is the combined orbital energy of both the electron and nucleus around their common center. The electron's electromagnetic radius is  $r_e = q/(4\pi\epsilon_0 m_e c^2)$  and  $q$  is the elementary charge.