CHAPTER 3

VELOCITY, ENERGY AND ACCELERATION

Velocity-Energy relationship

Inward acceleration

> How many of us are aware of the fact that we are racing through space 50 times faster than a rifle bullet at a speed of 30 km per second in our orbit around the Sun? We certainly can't feel it nor do we feel our velocity around the galaxy which is ten times faster. Furthermore, our velocity relative to the rest of the Universe, which equals or approaches *c,* the speed of light, and which is 1000 times faster yet, cannot be perceived except for the relativistic effects it generates as we observe in particle accelerators and in measurements involving fast atomic orbits.

3.1 Velocity-Energy relationship

It is not difficult to understand the meaning of velocity. Velocity is how far something moves in a specified unit of time. For example, a strong person might slam a tennis ball to a speed of 36 m per second which corresponds to an energy of $E = 36$ Joules if we use Newton's energy relation $E = \frac{1}{2}mv^2$ (assuming the tennis ball has a mass of $m = 0.055$ kg). Difficulties arise when we attempt to add velocities and energies because of their apparent non-linear relationship. Assume that the ball was hit by a tennis player in the forward direction while traveling on a train which is moving with a velocity of 36 m per second. To a stationary observer at ground the ball would, before it was struck, have the same velocity as the train and therefore an initial energy of $E = 36$ Joules. The additional energy ΔE of 36 Joules imposed by the player's racket makes the ball go 36 m/s relative to the player and the train but 72 m/s with respect to the observer at ground. Even though the player only added another 36 Joules of energy to the ball the stationary observer at ground records an increase of $\Delta E = 107$ Joules according to $E = \frac{1}{2}mv^2$. There is nothing wrong with the mathematics. The point is to show that when velocities are added to an already existing velocity, difficulties arise. The observer, which is at rest on Earth, might say that he is registering an "**absolute**" velocity (v_{abs}) and energy (E_{abs}) of the ball and that the energy differs by a substantial amount from the "**relative**" energy (ΔE) and velocity (Δv) registered by someone on the train. What if we take into account the Earth orbital velocity around the Sun? Careful examination will show that a hypothetical observer on the Sun would see an energy difference of the tennis ball that would differ drastically from the observer on Earth and as seen by the player and passengers on the train. The example of the tennis ball tells us that we are dealing with two types of energies, namely absolute energy and relative energy, as well as absolute and relative velocities.

The importance of absolute energy and relative energy can be further emphasized by the following example. Consider the absolute amount of energy $E_{abs} = 9\times10^{11}$ Joules required to launch a 2000 kg rocket to a velocity of $v_{abs} = 30$ km/s. Adding another 9×10^{11} Joules to the rocket while in flight, through the rocket's own propulsion system, will increase the velocity by a factor of two (disregarding the weight loss of spent fuel). However, had the extra 9×10^{11} Joules been added to the absolute energy at the launch site it would only have increased the velocity of the rocket by a factor of 1.4 or more exactly $\sqrt{2}$. Why the difference? In the first case relative energy is added on top of an already existing absolute energy *i.e.* the energy added in flight also includes the extra amount of energy the fuel itself had gained during the launch of the rocket, while in the second case the extra energy is simply increasing the amount of absolute energy at the launch. It is of utmost importance to note that energy and velocity can appear in both absolute and relative form. An observer outside our Universe will register energies and velocities as absolute energy and absolute velocity such as in Fig. 4 Chapt 2.2 and throughout this book absolute energy of matter and relative energy with respect to our frame of reference here on Earth will be distinguished as follows:

1. Absolute energy $E = (E_0 = m_0 c^2)$ and absolute velocity = υ_{abs}
2. Relative energy ΔE or ∇E, and relative velocity = $\Delta \upsilon$ and $\nabla \upsilon$

where E_0 is the absolute energy of matter at our frame of reference with respect to the center of mass of the Universe and ΔE is relative energy added to E_0 (gain in energy) and ∇E is relative energy subtracted from E_0 (loss of energy)

Normally we do not worry about the Earth's velocity of 30 km/s around the Sun or our velocity with respect to the rest of the Universe. However, at high velocities relative to us, such as found in particle accelerators and atomic orbits, our velocity relative to the rest of the Universe becomes very important (Mach's principle). Einstein's theory of relativity unfortunately, pays no attention to the above since it considers us at rest (thus the term rest mass energy $E_0 = mc^2$). Einstein's velocity-energy equation is usually written as

$$\Delta \upsilon = c\sqrt{1 - \frac{1}{\left(1 + \frac{\Delta E}{mc^2}\right)^2}}, \qquad (1/t) \qquad (2)$$

where $\Delta \upsilon$ is velocity of a mass m relative to us generated by the energy ΔE. Einstein's theory cannot deal with certain situations, for example where energy is lost, for the simple reason that velocity cannot be subtracted from rest or zero velocity. Equation (2) is only accurate in cases where rest mass energy is gained, such as in particle accelerators, but fails in cases where rest mass energy is lost, such as in atomic orbits where energy is lost to radiation (see Chapter 6, section 6.3). Einstein's theory can be said to be up against the same difficulties that our ancestors faced when trying to unravel the orbits of planets, believing that our Earth was at the center and at rest. Einstein's theory believes that we are at the center of the Universe and at rest. In fact it also believes that any observer on any other galaxy in the Univers can consider him or herself to be at the center and at rest.

By rewriting Einstein's Equation (2) (as later explained) and deriving at Equation (5), we discover that we actually are dealing with basic formulas of harmonic motion which demands that our frame of reference has to move with an absolute velocity of $v_0 = c$ relative to a central point $x = 0$, the center of mass of the Universe.

Our absolute velocity $v_0 = c$ relative to the rest of the Universe can be established through red-shift measurements. Since red-shifts of most distant galaxies (which by far outnumber ones that are close to us) show that recession velocities equal or approach the speed of light *c*, we can confidently assume that we are moving with *c* or very near *c* relative to the bulk of Universe rather than the whole Universe moving with *c* relative to us. Therefore, any relative velocity that we observe or experience here on Earth has to be added (or subtracted) by vector summation to our absolute velocity of $v_{abs} = c$. From the above analysis it is possible to construct a mathematical representation that nicely conforms with both Einstein's equation (2) and the equations of harmonic motion shown in Fig. 4, Chapter 2.2 and on page 146. The diagram in Fig. 4a demonstrates how we and our neighboring galaxies are accelerating toward the point $x = 0$, the center of mass of the system and marked along the x axis is our current position $x = x_0$. The circle surrounding $x = x_0$ represents the present horizon or limit of our largest telescopes (note that we can only see a very small fraction of the Universe). Also shown is the point of maximum amplitude where $x = A$, from which matter starts falling towards the center. At the maximum amplitude *A*, where potential or absolute energy of matter is infinite, the velocity is zero. Since Fig 4 indicates a loss in potential energy as matter falls with increased speed towards the center $x = 0$, where the absolute velocity has reached $v_{abs} = \sqrt{2}c$, we have an inverse velocity-energy relationship. At our position x_0 absolute velocity and absolute energy of matter are c and E_0 respectively. An increase or decrease in E_0 by ΔE or ∇E can be written in the form

$$v_{abs} = c\frac{E_0}{E_0 + \Delta E}, \quad \text{and} \quad v_{abs} = \sqrt{2}c - c\frac{E_0 - \nabla E}{E_0} \qquad (l/t) \qquad (3)$$

Equation (3), which yields absolute velocities at different points along the *x*-axis can also be derived from the trigonometric functions shown in Fig. 4. Absolute velocities produced relative to our frame of reference, cannot be added or subtracted linearly to our velocity c as Equation (3) might suggest. Relative velocities Δv or ∇v at our frame of reference x_0. For energy gained, the velocity relative to x_0 becomes

$$\Delta v = \sqrt{c^2 - v_{abs}^2}, \qquad (l/t) \qquad (4a)$$

and in cases where energy is lost

$$\nabla v = \sqrt{v_{abs}^2 - c^2}. \qquad (l/t) \qquad (4b)$$

For example, the velocity of a particle that has gained energy in a particle accelerator by a the amount of ΔE is therefore,

$$\Delta v = \sqrt{c^2 - \left(c\frac{E_0}{E_0 + \Delta E}\right)^2}, \qquad (l/t) \qquad (5)$$

which gives the same result as Einstein's Equation (2).

When ΔE is much smaller than E_0, Equation (5) can be reduced to

$$\Delta v \approx \sqrt{c^2 \frac{2\Delta E}{E_0}}, \qquad (l/t) \qquad (6)$$

and replacing E_0 with Einstein's $E_0 = m_0 c^2$ leads to Newton's familiar

$$v \approx \sqrt{\frac{2E}{m_0}} \text{ or } E \approx \tfrac{1}{2} m_0 v^2, \qquad (ml^2/t^2) \qquad (7)$$

where m_0 is mass of matter at relative rest with respect to x_0.

Although the above Equation (5) produces the same result as Einstein's Equation (2) from his Special Theory of Relativity, it contradicts the concept of relative rest because it postulates that our frame of reference must be moving with an absolute velocity of *c* relative to a common center, the center of mass of the Universe. The inverse energy-velocity relationship and the vectorial addition of velocities are also demonstrated by the diagrams in Fig. 5a and b, where velocities are shown as a function of distance from the center of mass of a gravitating system. Fig. 5a shows the Universe as whole and Fig. 5b illustrates the relationship between velocity and orbital radius of our Earth and the planet Mars.

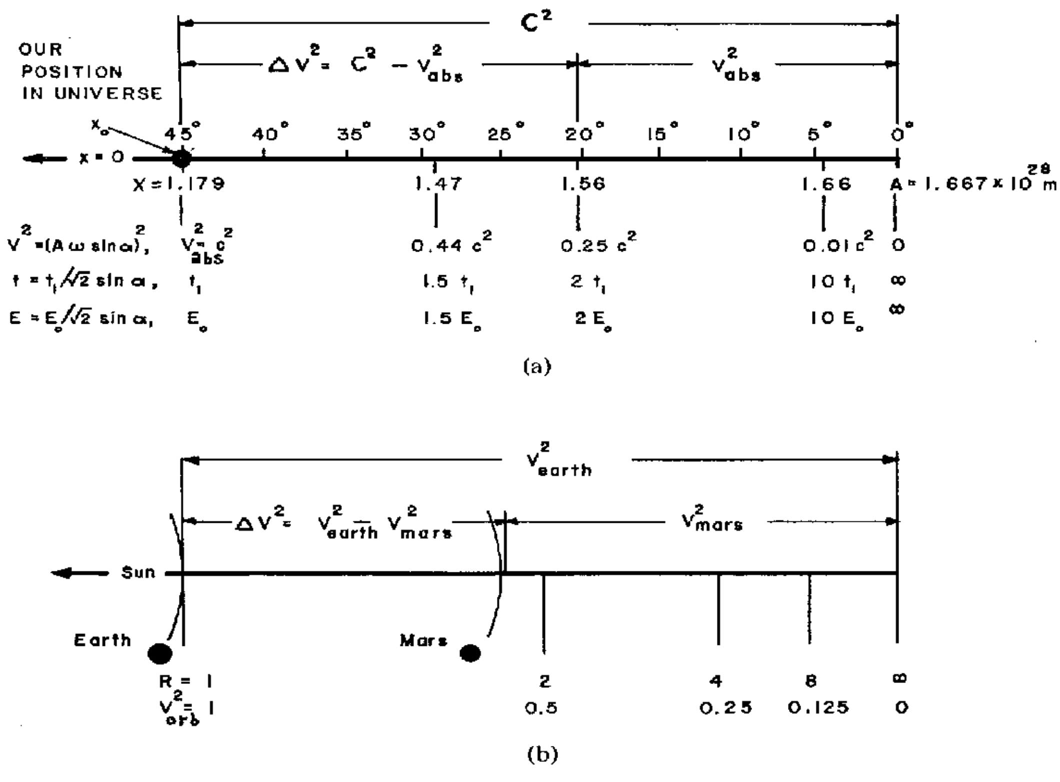

Fig. 5a. Diagram showing the relationship between energy, absolutevelocity and time along the x-axis of the Universe. b. Diagram showing the relationship between absolute velocity and orbital radius in our planetary system. (Radius and velocity in Earth units).

In the diagram of Fig. 5b it can be seen that Mars, which is at a higher gravitational potential energy level than Earth, has a lower orbital absolute velocity.

As previously stated, it appears that from our limited view of the Universe, we and all galaxies surrounding us, are hurling toward a distant point, too far to observe. This point is the center of mass of the system at $x = 0$ in Fig. 4. Chapt.2.2, Galaxies ahead of ours, which have accelerated farther and are falling faster towards $x = 0$, seem to speed away from us. By the same token we appear to pull away from galaxies that are behind us and still in an early state of fall. For an observer at x_0 it creates the illusion that the whole observable Universe is pulling apart, or expanding, while in reality it is collapsing. Galaxies at the other side of the center, are they not coming towards us? The answer is that we cannot see that far. Our present technology allows us to see only about one percent of the distance to the center of the Universe which means that only about one millionth of the total volume of the Universe is observable to us x_0 assuming a Universe with spherical shape.

It should be noted that from our frame of reference at x_0 the maximum velocity of other astronomical objects is *c*, the speed of light. This is because we are already traveling at *c* so when looking back to the maximum amplitude *A*, where matter is at zero velocity, the observed velocity difference must be *c*. To reach a point at *A* or zero velocity would require an infinite amount of energy according to Equation (5). Looking towards the center $x = 0$ where the absolute velocity of matter is $\sqrt{2}c$, according to the simple model in Fig. 4, Chapt. 2.2 the velocity relative to us again appears as c according to the vector sum

$$\nabla v = \sqrt{2c^2 - c^2} = c. \qquad (1/t) \qquad (8)$$

Note that Equation (8) refers to absolute velocities of Galaxies ahead of ours, which are closer to the center of mass $x = 0$, that have lost potential energy and are falling faster toward the center. Relative velocities that are produced by loss of potential energy are denoted ∇v. Another example of velocities produced by loss of potential energy are velocities of electrons captured in atomic orbits where potential energy also is lost to radiation.

Velocities relative to us generated by loss of energy are obtained by

$$\nabla v = \sqrt{c^2 - (2c^2 - v_{abs}^2)} \quad , \qquad (l/t) \qquad (9)$$

or

$$\nabla v = \sqrt{c^2 - \left(c\frac{E_0 - \nabla E}{E_0}\right)^2} \, , \qquad (l/t) \qquad (10)$$

where ∇E is potential energy lost relative to our frame of reference at x_0. Equation (10) differs drastically from Equation (5) and Einstein's Equation (2) but its validity is clearly demonstrated in Chapter 6, section 6.3, which deals with the subject of atomic orbits.

3.2 Inward acceleration

Both Hubble's law and the Uncertainty Principle specify a change in velocity Δv with a simultaneous change in distance or position Δx, and the only mechanism that can cause such changes is acceleration.

Slipher (1917) discovered that light from galaxies are red-shifted and concluded that most galaxies must recede from us with high velocities. For example, the Coma cluster of galaxies, 100 Mps $= 3.09 \times 10^{24}$ m or 3.26×10^{8} light years away seems to recede with a velocity of 6.86×10^{6} m/s. The discovery by Slipher of cosmic red-shifts inspired Hubble (1929) to complete more measurements which led him to formulate a law of recession $H = \Delta v / \Delta x$ where Δv is the velocity of recession relative from us at Earth and Δx the distance of separation between us and other galaxies. Hubble argued that if red-shifts of distant galaxies are caused by the Doppler effect, where light waves stretch and get longer or redder as galaxies recede, then the red-shifts would indicate how fast individual galaxies separate from each other.

The fact that galactic red-shifts are observed in all directions and increase with distance motivated Hubble and many others to believe that the Universe is expanding in a linear fashion as described by Hubble's law $H = \Delta v / \Delta x$. It is unfortunate that Hubble's law was misinterpreted as an expansion law when in reality it has the physical dimensions of angular frequency. The problem is, that in a scenario where everything is expanding, by change in velocity with distance, the expansion law must include acceleration. Instead of writing Hubble's law as a change in velocity per distance it could perhaps have been better stated as a change in velocity per light-year or $H = \Delta v / \Delta t$, (acceleration) where Δt is light years.

Slipher's and Hubble's discoveries clearly show that galaxies separate with velocities that increase with distance, but since distances between galaxies also change with time their velocities of separation must also change with time. From basic physics we know that any change in velocity with time equals acceleration. The expansion scenario therefore, implies that all galaxies and all matter contained within them, are subject to a cosmic acceleration which will be identified as a_0 (Wåhlin (1981)). The rate of a_0, which is very small, can be derived from Hubble's parameter and using the above velocity and distance values for the Coma cluster we obtain a value of

$$a_0 = \frac{\Delta v}{\Delta t} = \frac{(\Delta v)^2}{2\Delta x} \approx 7.62 \times 10^{-12} ms^{-2} . \qquad (l/t^2) \qquad (11)$$

It is understandable why Hubble and many other astronomers have failed to recognize the presence of acceleration in their observations, since the star filled sky always seems motionless to an observer, like a still picture, and there is no apparent change in velocity or position over any period of time making it practically impossible to visually detect any acceleration.

If our Galaxy, including all matter contained within it, is subject to a cosmic acceleration a_0, it should be possible in some way to detect a small change in velocity and displacement of particles in our laboratory caused by such an acceleration. In fact, such changes are predicted by the Uncertainty Principle. In the laboratory therefore, we should be able to determine the rate of acceleration very accurately from the Uncertainty Principle which postulates that an electron must change velocity momentum Δp and position Δx according to the following equation

$$\Delta p \Delta x = \tfrac{1}{2}\hbar, \qquad (ml^2/t) \qquad (12)$$

where $\hbar$ is Planck's constant divided by 2π or $\hbar = h/(2\pi)$.

Planck's constant was discovered by Max Planck (1900) who found that electrons in atoms oscillate and radiate energy at fixed frequencies of ω where the energy per ω always equals $\hbar$. The energy of an electron, which is the smallest quantized form of matter, can therefore be expressed in frequency by way of Planck's constant or $E_e = \omega_e \hbar$. This remarkable property proves that a particle such as the electron is oscillating and has a wavelength. Planck's wavelength for an electron is $\lambda_e = 2\pi\hbar c / E_e$ which is also called the electron's Compton wavelength.

Since Δp and Δx in Equation (12) are inversely proportional and their product equals a constant (Planck's constant) we can also write

$$p_{\max} x_{\min} = \tfrac{1}{2}\hbar, \qquad (ml^2/t) \qquad (13)$$

where $p_{\max}$ is the electron's maximum momentum and $x_{\min}$ the electron's corresponding minimum displacement. Choosing the maximum or rest mass momentum of an electron $p_{\max} = m_e c$ (from Einstein's relation $E_e = m_e c^2$ or v_{abs} at our reference point x_0 in space) permits us to obtain the associated displacement $x_{\min}$ as determined by $\hbar$. Since $\hbar$ is in units of angular frequency $\omega_1 = 2\pi / s$ or one radian per second we can write $x_{\min} = a_o / \omega_1^2$ and Equation (13) becomes

$$\frac{m_e c a_0}{\omega_1^2} = \tfrac{1}{2}\hbar. \qquad (ml^2/t) \qquad (14)$$

Solving for a_0 yields an acceleration of $7.622479 \times 10^{-12}\,\mathrm{ms}^{-2}$ which is the same value as that obtained from Hubble's law Equation (11). However, a different approach, which avoids the unconventional term $p_{max} = m_e c$, is to write the Uncertainty Principle as

$$\Delta E_e \Delta t = 2\pi h, \qquad (ml^2/t) \qquad (15)$$

and at small non-relativistic velocities

$$\tfrac{1}{2} m_e (\Delta v)^2 \Delta t \approx 4\pi^2 \tfrac{1}{2}\hbar. \qquad (ml^2/t) \qquad (16)$$

Velocity of matter can only change as a function of time and without violating the uncertainty principle we can divide both sides by $(\Delta t)^2$ and write

$$\frac{\tfrac{1}{2} m_e (\Delta v)^2}{\Delta t} \approx \frac{4\pi^2 \tfrac{1}{2}\hbar}{(\Delta t)^2}, \qquad \text{(power)} \qquad (ml^2/t^3) \qquad (17)$$

since $4\pi^2/(\Delta t)^2 = \omega_1^2$ we can substitute and change Equation (17) to

$$\frac{(\Delta v)^2}{\Delta t} \approx \frac{\omega_1^2 \hbar}{m_e}. \qquad (l^2/t^3) \qquad (18)$$

When we observe red-shifts from the Coma cluster, 3.26×10^8 light-years away, we are also looking back in time. Inserting a value of $\Delta t = 3.26 \times 10^8$ light-years (1.028×10^{16} s) in Equation (18) above gives a velocity difference of $\Delta v = 6.86 \times 10^6 ms^{-1}$ which again agrees with observations and the value used in Equation (11). The above results lead to the conclusion that both Hubble's observations and the Uncertainty Principle are a direct consequence of the cosmic acceleration a_0. One major problem when using Hubble's law is that Hubble's constant H has not yet been precisely determined (van den Bergh (1981)). Hubble's constant also seems to vary with celestial latitude (de Vaucouleurs (1978)) and it was never clear from observations if Δv increases linearly with Δx (Soneira (1979)) or follows a quadratic relationship as shown here (see Chapter 7 sec. 7.1) and which has been suggested by other investigators (Karachentsev

(1967), Ozernoy (1969), de Vaucouleurs (1971) and Segal (1980)). The Coma cluster, for which over 800 red-shift measurements are available, is perhaps one of the best candidates for determining the cosmic acceleration a_0. Also, the quadratic and linear relationships both predict about the same red-shift for the Coma cluster (see Fig. 15, Chapter 7). A large scale quadratic function might appear linear on a small local scale which is probably why most astronomers came to believe that the expansion is linear. The rate of a_0 turns out to be equal to the classical electrons surface acceleration due to its gravitational field or $a_0 = Gm_e / r_e^2$, where $r_e = q^2 \mu_0 / (4\pi m_e)$ is the electron's electromagnetic radius and μ_0 the permeability constant. This relationship, which may not be a coincidence, will be discussed in Chapter 8.

Once the velocity and rate of acceleration are known, numerous other parameters of the harmonic Universe can be obtained. Most important is the loss in potential energy and inertia of mass of matter as it accelerates towards the cosmic center $x = 0$. We know from laboratory experiments that acceleration or more precisely deceleration causes matter to radiate energy. The heating of a bullet stopped by a brick wall proves this point. High velocity electrons radiate x-rays when stopped by metal targets (anodes) in x-ray tubes where the slowing or deceleration of high velocity electrons creates electromagnetic radiation referred to as bremsstrahlung (braking radiation). All the above cases show that loss of potential energy results in radiation. In the collapsing Universe matter loses potential energy at the rate of $ma_0c/2\pi$ (watts) due to the steady cosmic acceleration a_0. An electron, therefore, continuously radiates a minute amount of power equal to $L_e = m_e a_0 c / 2\pi = \frac{1}{2}\ddot{h}$ (watts) where $\ddot{h} = h/s^2$ is Planck's constant expressed in power. The term 2π distributes the power over one cycle and $\frac{1}{2}\ddot{h}$ is also the observed amount of power radiated in the electron's so called zero point energy state. As mentioned before, it is the cosmic acceleration a_0 and subsequent change in potential energy of matter that makes the "world go around". If there was no acceleration present there would be no change in energy and nothing would ever happen. It must be remembered that inertial mass is also a measure of energy (Einstein's $E = mc^2$) so as matter loses potential energy it also loses inertial mass. When inertial mass, or simply mass, converts into radiation it produces electromagnetic waves which again can revert back to mass through collision processes. When a gun is fired the burning powder in the cartridge loses mass to heat and electromagnetic radiation, which in turn will add mass to the bullet as it is propelled to a higher velocity and a higher energy state.

So far the terms acceleration and deceleration have been used quite loosely. Why is it that in a collapsing Universe matter loses energy as it accelerates and gains velocity while here on Earth we seem to observe the opposite that energy increases when matter accelerates and gains velocity? From our view point at Earth, it requires additional energy to accelerate a projectile to a given velocity and this energy becomes lost when the projectile decelerates back to a stand-still again. The discrepancy has to do with the paradoxical behavior of harmonic motion where the question of acceleration and deceleration depends on the frame of reference. An astronaut in an earth orbiting spacecraft has to fire booster rockets and add energy to the space craft in order to accelerate out to a higher orbit. An observer on Earth however, would see the astronaut decelerate since the spacecraft in reality moved from a lower and faster orbit to a higher but slower orbit. The opposite is true when retrorockets are fired in order to slow down the spacecraft for reentry because now the earthbound observer will see the astronaut accelerate and speed up as the spacecraft spirals nearer Earth in ever closer but faster orbits. To summarize, an outside observer finds the spacecraft decrease in velocity as a function of added energy while the opposite seems to occur for the astronaut traveling with the spaceship. To an outside cosmic observer therefore, matter in a collapsing Universe will lose energy as it accelerates and gains velocity and conversely slow down when it gains energy. There appears to be a constant interchange between potential energy and radiation in nature as a result of acceleration and deceleration. We know that inertial mass can convert to radiation and likewise, radiation can change back to inertial mass as matter absorbs radiation. One interesting question is: do we live in a Universe where just as much mass is being created as is being radiated? *i.e.* is there an equilibrium or perfect balance between energy of matter and energy of radiation in the Universe? In the chapters that follow we will find evidence in favor of a matter-radiation balanced Universe.

Another interesting effect, first discovered by Zwicky in 1937, is that galaxies and cluster of galaxies seem to have more rotational speed than can be accounted for by their own visible gravitational mass. This missing mass problem can possible be explained by the tidal force and velocity that the cosmic acceleration a_0 will generate over the large cosmic distances occupied by galaxies. The amount of total rotational velocity with the added tidal effect of a_0 can be as much as

$$v = \sqrt{\left(GM_{gal} / R_{gal}\right) + \left(4a_0 R_{gal}\right)}$$

where R_{gal} is the radius of a galaxy and M_{gal} its visible mass within R_{gal}. In simple terms, Hubble's expansion not only increases the velocity between galaxies it also increases the velocity of stars within the galaxies themselves.