CHAPTER 8

LARGE NUMBER HYPOTHESES AND THE VIRIAL THEOREM

A large number such as 10^{42} or 10^{85} might not appear monumental. It is easy to write down and compared to infinity it is insignificant. But how many of us remember the inventor of chess who, when offered a reward of his own choice, asked the maharajah for a grain of rice on the first square, two on the next and four on the third and so on until all sixty-four squares on the board had been filled. “Is this all you ask?” said the maharajah not realizing it would take more than 10^{19}grains of rice to accomplish this. In fact the amount of rice would cover all India with a foot or 30 cm layer of rice. Had he asked for 10^{85} rice grains it would have filled a spherical volume 10 billion light-years across.

8.1 Large number ratios

Ever since Herman Weyl (1919) come across the large numerical ratio of 4×10^{42} there has been a great interest in trying to connect it with other number ratios that appear in nature having a similar magnitude. Weyl derived at his large number ratio in a roundabout way by comparing the classical or electromagnetic radius of the electron $r_e = q_e^2 / (4\pi\varepsilon_0 m_e c^2)$ to the hypothetical radius of a particle with the same charge q_e having an electrostatic energy equal to that of the electron’s gravitational energy. This ratio turns out to be the same as the ratio between the electron’s electrostatic coulomb force F_e to its gravitational force F_g or the energy E_0 of the electron’s electrostatic field to the electron's self energy due to its gravitational field E_g. Weyl further speculated that the above large ratio might also be the ratio between the radius of the Universe and the radius of the electron. In fact some years later in 1931 the astronomer John Q. Stewart pointed out that the radius of the Universe from Hubble's law, and Einstein's relativity theory, divided by the radius of the electron, comes within a factor of one hundred of Weyl's ratio.

In the same year Arthur S. Eddington (1931) speculated that the ratio of the electrostatic force to the gravitational force between an electron and a proton equals the square-root of *N,* the number of particles in the Universe. For reasons unknown Eddington thought that *N* had to be taken as the number of protons rather than electrons.

Up to the present time the general view has been that the large number ratios which appear in nature are more or less a coincidence rather than a serious natural relationship. In the pages that follow, it will be shown that Weyl's notion and Eddington's speculation are in fact true manifestations of nature because both Weyl's large number ratio and Eddington's magic number fit precisely the harmonic model of our Universe described in this book. An attempt will be made to find the connection between Weyl's and Eddington's large numbers and the mechanical structure of the Universe.

8.2 Eddington's magic number $N = 1.7507 \times 10^{85}$

Since the proton is found unstable we will duplicate Eddington's arguments, the only difference being that *N* will represent the number of electrons and positrons in the Universe and that $\sqrt{N}$ will equal the ratio of the electrostatic to gravitational forces, or energies between two such particles as suggested by Weyl. In Eddington's *The Expanding Universe* (1923) one can follow the reasoning that led to his large number hypothesis and the basic equations underlying his theory are

$$\frac{GM_u}{R_u} = c^2, \qquad (l^2/t^2) \qquad (97)$$

and the ratios between the electron's electrostatic and gravitational forces and energies

$$\frac{F_e}{F_g} = \frac{E_0}{E_g} = \frac{q^2/(4\pi\varepsilon_0 r_e)}{Gm_e^2/r_e} = \sqrt{N}\,, \qquad (none) \qquad (98)$$

which equals Weyl's number. In the above equations R_u is the radius of curvature of the Universe; M_u its mass; *c* the maximum speed of recession of distant objects; *G* the gravitational constant; *N* the number of particles of mass m_e in the Universe; *q* the electron's charge; and $r_e = q^2/(4\pi\varepsilon_0 m_e c^2)$ is the electrostatic or classical radius of an electron (the proton's mass has purposely been replaced with the electron's or positron's mass m_e). From Equation (98) it immediately follows that $N = 1.7507\times10^{85}$ and the total mass of the Universe becomes

$$M_u = Nm_e = 1.5948\times10^{55}\,\mathrm{kg}. \qquad (m) \qquad (99)$$

The radius of curvature from Equation (97) is then

$$R_u = \frac{GM_u}{c^2} = r_e\sqrt{N} = 1.17908\times10^{28}\,\mathrm{m}, \qquad (l) \qquad (100)$$

and the mean mass density of the Universe equals

$$\rho = \frac{M_u}{\frac{4}{3}\pi R_u^3} = \frac{H^2}{\frac{4}{3}\pi G} = 2.3227\times10^{-30}\,\mathrm{kg\ m^{-3}}. \qquad (m/l^3) \qquad (101)$$

In the above equation, *H* is Hubble's constant. Hubble's constant, which has the dimensions of angular frequency, relates to the fundamental frequency ω_0 of the Universe or

$$\omega_0 = \frac{c}{R_u} = H = 2.5425\times10^{-20}\,\mathrm{rad\ s^{-1}}, \qquad (t^{-1}) \qquad (102)$$

which corresponds to a time epoch or period of

$$t_0 = \frac{2\pi}{\omega_0} = 2.4712 \times 10^{20}\,\text{s}. \qquad (t) \qquad (103)$$

Further calculations show that the gravitational acceleration at R_u equals

$$a_0 = \frac{GM_u}{R_u^2} = \frac{Gm_e}{r_e^2} = 7.6225 \times 10^{-12}\,\text{m s}^{-2}. \qquad (l/t^2) \qquad (104)$$

The significance of the large number hypotheses becomes clear when we compare the above results with the physical constants and parameters listed in Table 1, Chapter 2, section 2.2. Perhaps the most important feature revealed by the above equations is that the electron's gravitational potential energy at R_u, generated by the rest of the Universe, equals the electron's rest mass energy and electrostatic energy

$$E_0 = \frac{GM_u m_e}{R_u} = \frac{q^2}{4\pi\varepsilon_0 r_e} = m_e c^2. \qquad (ml^2/t^2) \qquad (105)$$

We can therefore change expression (98) by replacing the electron's electrostatic energy $q^2/(4\pi\varepsilon_0 r_e)$ with the potential energy $GM_u m_e / R_u$ and write Weyl's number to equal

$$\frac{E_0}{E_g} = \frac{GM_u m_e / R_u}{Gm_e^2 / r_e} = 4.183 \times 10^{42}, \qquad (none) \qquad (106)$$

from which other natural ratios and the square root of Eddington's magic number can be obtained

$$\frac{R_u}{r_e} = \sqrt{\frac{M_u}{m_e}} = \sqrt[3]{\frac{M_u R_u}{m_e r_e}} = \sqrt{N} = 4.184 \times 10^{42}. \qquad (none) \qquad (107)$$

It is not known whether Weyl or Eddington arrived at their hypotheses by intuition or by considering the substitution described in Equation (107). Another interesting consequence of the large number ratios is the ratio of the electron's gravitational self-energy to its smallest quantized energy (see Chapter 5, section 5.6) due to the fundamental frequency of the Universe equals

$$\alpha = \frac{Gm_e^2 / r_e}{\omega_0 \hbar} = \frac{1}{137.036}, \text{ (finestructure constant)} \quad (none)(108)$$

where $\hbar$ is Planck's constant h divided by 2π. This is the so-called fine structure constant which was of special interest to Eddington,

From the laws of harmonic motion we can also determine the electron's change in potential energy (radiation) as a function of time

$$\frac{\nabla E}{\Delta t} = \frac{m_e c^2}{t_0} = \frac{m_e a_0 c}{2\pi} = L_e, \qquad (ml^2 / t^3) \quad (109)$$

and

$$L_e = \tfrac{1}{2} \ddot{h}, \qquad (ml^2 / t^3) \quad (110)$$

where L_e is the power radiated by an electron generated by the harmonic motion of the Universe and $\ddot{h}$ is Planck's constant expressed in power. From Equations (104), (109) and (110) follows the ratio

$$\frac{G}{\ddot{h}} = \frac{r_e^2 \pi}{m_e^2 c}. \qquad (lt / m^2) \quad (111)$$

Since Plank's constant is accurately determined from experiments we can derive $G = 6.6445 \times 10^{-11}$ as the postulated free space value of the universal gravitational constant. This is about 0.004 times less than the current estimated value obtained at the Earth's surface. There are however, both theoretical (Fujii (1971) and Long (1980)) and experimental (Long *et al.* (1976) and Stacey *et al.* (1981)) claims indicating a small departure, at the Earth's surface, from the astronomical or free space value of G, which could explain the above discrepancy. It should be mentioned that the universal gravitational constant G is one of the least well known constants in nature and the method of obtaining G has often been criticized (Kazuaki Kuroda (1995)).

8.3 Ampère's equation

It is difficult to ignore the implications of Weyl's and Eddington's large number hypotheses. The large number hypotheses has intrigued many theoretical physicists and most noteworthy P.A.M. Dirac (1938). Dirac, who unfortunately did not refer to Weyl's or Eddington's work, also compared other large numbers with the electron-proton electric to gravitational force ratio.

The amazing results of the large number hypotheses is that they seem to offer a link between electromagnetic and gravitational forces. The same is true for Ampère's law of electrodynamics which hints to the fact that c, the velocity of matter due to the harmonic motion of the Universe, is responsible for the electron's Coulomb force and that its gravitational force is generated by the cosmic acceleration a_0:

1. The force, or glue, that holds the electron together preventing it from flying apart due to its own charge is brought about by the electromagnetic "pinch" effect generated by its velocity of c in space. The magnitude of this force equals $q^2/(4\pi\varepsilon_0 r_e^2) = m_e c^2 / r_e$.

2. The gravitational energy of an electron seems to originate from the "pinch" effect generated by the cosmic acceleration a_0 and the magnitude of this energy is $m_e a_0 r_e$.

The pinch effect originates from electromagnetic forces created by a moving charge. The electromagnetic force, which points inward towards the center of a moving charge, tends to squeeze the charge together. The magnitude of this force can be determined by Ampère's equation (1820). This equation was first derived from the observation that a force appears between two parallel cylindrical conductors that carry currents of electric charge. If the currents through both conductors flow in the same direction the force is attractive and if the currents flow in opposite directions the force is repulsive. The diagram in Fig. 16 shows Ampère's experiment consisting of two parallel conductors carrying currents i_a and i_b respectively. The force between the conductors is given by

$$F = li_b B_a = l \frac{i_a i_b \mu_0}{2\pi \quad R}, \qquad (ml/t^2) \qquad (112)$$

where l is the length of the current elements which are separated by the distance R. The magnetic field generated by one current at the site of the other is B.

If we consider the current elements as individual charges $(i = q/t)$, such as single electrons moving at a velocity of c, Ampère's formula becomes

$$F_e = \frac{l}{t^2} \cdot \frac{q_a q_b \mu_0}{2\pi \quad R} \cdot \left(\frac{r_e}{R}\right), \qquad (ml/t^2) \qquad (113)$$

where $l = 2r_e$ is the length of the current element or in this case

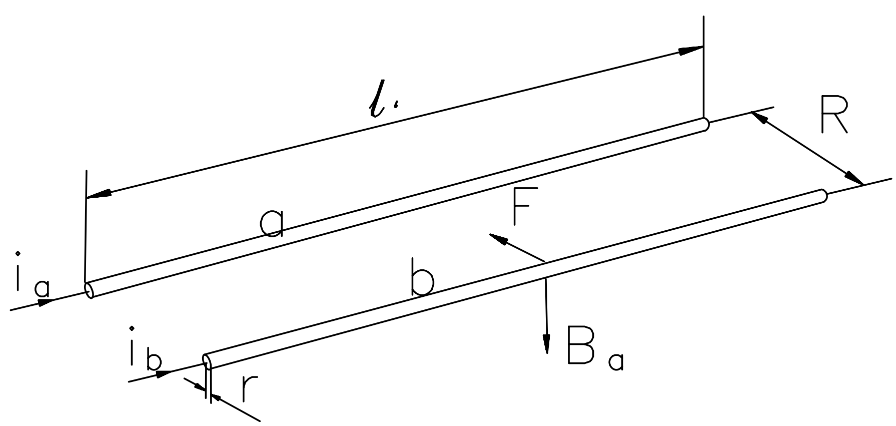

Fig. 16. Two parallel wires which carry current in the same direction attract each other.

the size of an electron and $t = 2r_e / c$ is the time required for the electron to move the distance l. The term (r_e / R) changes Ampère's equation from cylindrical symmetry of conductors to the spherical geometry of single electrons. So far we have discussed the force between conductors or two electrons moving side by side immersed in each others magnetic fields. Since a moving electron is also immersed in its own magnetic field it will be subject to a self-induced pinching force that tends to squeeze it together according to Equation (113) thus counteracting the electron's own Coulomb force which wants to blow it apart. In fact, Equation (113) can easily be converted to Coulomb's force equation by replacing t with $2r_e/c$ and μ_0 with $1/(\varepsilon_0 c^2)$ which results in

$$F_e = \frac{q_e^2}{4\pi\varepsilon_0 R^2} \text{ (Coulomb's law).} \qquad (ml/t^2) \quad (114)$$

There is also a force F_g superimposed on charges that are mutually accelerated which according to Ampère's law (113) can be written as

$$F_g = \tfrac{1}{2}a \cdot \frac{q_a q_b \mu_0}{2\pi \quad R} \cdot \left(\frac{r_e}{R}\right), \qquad (ml/t^2) \quad (115)$$

l/t^2 in (113) is replaced by $\tfrac{1}{2}a,$ were a represents acceleration. Inserting a_0, the acceleration due to the harmonic motion of the Universe, makes Equation (115) identical Newton's law of gravitation

$$F_g = a_0 \cdot \frac{q_a q_b \mu_0 r_e}{4\pi \quad R^2} = \frac{G m_a m_b}{R^2} \text{ (Newton's law),} \quad (ml/t^2) \quad (116a)$$

where m_a and m_b are the masses of two electrons separated by a distance R. Equation (116a) might prove that the agency responsible for generating gravitational attraction is the cosmic acceleration a_0.

From the above analysis we find that Ampère's equation (113) predicts that an electron moving with a velocity of c generates a pinch force on itself by being immersed in its own magnetic field B. The force due to this pinch effect is opposite and equal to the electron's own Coulomb force and thus serves as the force binding the electron together. We have also seen that a similar pinch effect appears when an electron is subject to acceleration as shown by Equation (116a) which can also be written as

$$F_e = a_0 \frac{q_e^2 \mu_0}{4\pi r_e} = m_e a_0 . \qquad (ml/t^2) \quad (116b)$$

This leads to the fact that $m_e = q_e^2 \mu_0 /(4\pi r_e)$, or that the electron's inertia of mass is equivalent to the electron's inertia of charge (self-induction). What if two massive objects are accelerated relative to our laboratory, does the gravitational force between them increase in accordance with Equations (116a and 116b)? The answer is probably yes but we must first convert relative acceleration to absolute acceleration similar to that of relative and absolute velocities as described in Chapter 3, section 3.1. The increase in gravitational force between two objects being accelerated by a given acceleration a relative to our frame of reference should (without proof) be equal to

$$F \cong \left[1 + \frac{a^2}{2c^{\bullet} a_0} \right] \times \frac{G m_a m_b}{R^2} , \qquad (ml/t^2) \qquad (117)$$

where $c^{\bullet} = c$ per second. The gravitational force generated should be attractive for bodies decelerating relative to our frame of reference.

In Equations (116a and 116b) only the smallest amount of quantized matter were considered, namely that of the electron and the positron. However, there should be no problem with other particles, charged or neutral, if we assume that all matter is made up of electrons and positrons as suggested by Eddington's magic number *N*.

Ampère's formula gives only the magnitude of force between moving charges and not the direction as to whether attractive or repulsive. Also, the gravitational force of Equations (116a, 116b and 117) is believed to be of monopole nature and cannot be shielded against.

The cosmic structure revealed by the large number ratios is that of an oscillating Universe (expanding-contracting) in which the maximum speed of recession c can be interpreted as our velocity relative to distant matter or to the center of mass of the Universe. The frequency of oscillation is $\omega_0 = a_0 / c$ and a_0 represents the cosmic acceleration associated with the harmonic motion. The cosmic acceleration a_0 described in Chapter 3, section 3.2, is in fact evident from the discovery and observation that galaxies separate with velocities that increase with distance of separation. Since the distance of separation increases with time, the velocity must also increase with time. An increase in velocity with time is acceleration.

The large number hypotheses are, in the author's opinion, not just a coincidence but a direct result of the laws of harmonic motion where the energy or force between matter is balanced about a common center, the center of mass of the system. For example, if we compare the electron's gravitational self energy Gm_e^2/r_e to the rest mass energy generated by the rest of the Universe $GM_u m_e/R_u$, the following ratios appear

$$\frac{GM_u m_e/R_u}{Gm_e^2/r_e} = \frac{R_u}{r_e} = \sqrt{\frac{M_u}{m_e}} = \sqrt{N}. \qquad (none) \quad (118)$$

Since GM_u/R_u equals c^2 we can also write

$$\frac{c^2}{Gm_e/r_e} = \frac{E_0}{E_g} = \frac{F_e}{F_g} = \frac{\phi_u}{\phi_e} = \sqrt{N}, \qquad (none) \quad (119)$$

where E_0 is the electron's rest mass energy and E_g its gravitational self energy. One important point here is that the electron's rest mass energy $E_0 = m_e c^2$ is equal to the electron's electrostatic energy $E_0 = q^2/(4\pi\varepsilon_0 r_e)$ which implies that both energies are of the same origin or one electron would otherwise carry double the energy, which is not the case as shown from electron inhalation studies. Electron inhalation is a phenomenon where a positive charged electron (positron) is absorbed by a negative charged electron and where both particles mass and charge disappear into radiation in the form of two photons, each photon carrying an energy of E_0. The large number ratios can also be derived from the Virial Theorem which in the author's opinion, again proves that the large number ratios are not a mere coincidence but are simply in compliance with the laws of Nature.

8.4 The Virial Theorem and Cosmology

The Virial Theorem was originally applied to thermodynamics and statistical mechanics. It is now a very important tool in all branches of physics because it provides a means for checking the validity of both experiment and theory. Most cosmological models of today fail the virial test creating a missing mass problem and the purpose here is to test our cosmic model based *in toto* on the Virial Theorem. The result again leads to a relationship between G, the Universal gravitational constant, and h, Planck's radiation constant.

The Virial Theorem is the creation of Rudolf Julius Emanuel Clausius, 1822-1888, who was first to apply the doctrine of probabilities, in a systematic way, to the kinetic theory of gases and he also introduced the concept of entropy which is another important contribution made by him to science. The Virial Theorem states that the virial of a body or a large assembly of bodies in a confined space is defined to be the sum

$$\sum = \tfrac{1}{2}\mathbf{r}_i\mathbf{F}_i\,, \qquad (ml^2/t^2) \quad (120)$$

where $\mathbf{r}_i$ and $\mathbf{F}_i$ are the position and force vectors respectively, acting on the *ith* particle. In Latin virial means strength or stress, but in scientific terms we can describe it as half the product of the stress due to the attraction or repulsion between two particles in space multiplied by the distance between them, or in the case of more than two particles half the sum of such products taken for the entire system. In simple language, the Virial Theorem tells us that the stress or potential energy inside a given volume is balanced by the kinetic energy of matter residing within that same volume or

$$\tfrac{1}{2}m\frac{U}{\rho} = \tfrac{1}{2}m\overline{v}^2\,, \qquad (ml^2/t^2) \quad (121)$$

where $U = E/V$ (energy density) is the energy per unit volume, $\rho = m/V$ (mass density) is the amount of matter per unit volume and $\bar{v}$ is the velocity of a particle or the root mean velocity of particles with mass m. The potential energy might be electrostatic, binding molecules together, or it could be the gravitational energy which holds our solar system or entire Universe together.

The usefulness of the Virial Theorem can be demonstrated in several ways. For example, reducing Equation (121) to

$$\frac{U}{\rho} = v^2, \qquad (l^2/t^2) \quad (122)$$

implies that energy density divided by matter density relates to a fixed velocity squared. If we insert the energy per unit volume as equal to the molecular binding energy in a metal (stress modules or Young's modules), and divide it by the matter density, we can solve for v, the wave velocity of sound in the metal.

When applied to the electrostatic field produced by one or n number of electrons and positrons, Equation (122) becomes

$$\frac{U}{\rho} = \frac{nq^2}{4\pi\varepsilon_0 r_e V} \cdot \frac{V}{nm_e} = c^2, \qquad (l^2/t^2) \quad (123)$$

where q and m_e are the electron's or positron's charge and mass respectively, and $r_e = q^2/(4\pi \ \varepsilon_0 c^2)$ is the electrostatic or classical radius of an electron and positron. The symbol V in Equation (123) represents the volume in which n electrons or positrons are contained. The result is $v = c$, the velocity of light, or the speed at which electromagnetic waves and field changes propagate through space. Since U/ρ, which represents energy per mass, is equal to c^2 which in turn equals the cosmic gravitational tension ϕ_{univ} we can rewrite the Virial Theorem as follows:

$$\phi_{Univ} = \frac{E}{m} = c^2, \qquad (l^2/t^2) \quad (124)$$

which leads to Einstein's energy relation $E = mc^2$ (see Appendix C).

The Virial Theorem is used extensively in astrophysics and cosmology and it is in this branch of physics where it is often disobeyed. There is no apparent problem locally, such as within our own solar system. It is at large extra-galactic distances where observation and most present theories drastically deviate from the Virial Theorem and it worsens the larger and farther away these systems are, as was discovered many decades ago by F. Zwicky (1937). Zwicky found that the apparent mass of the Coma cluster is many times larger than the combined mass of all its individual galaxies, thus creating a missing mass problem (see Chapter 3, p48).

Most cosmological models suffer the same difficulty since they predict a mean mass density that is orders of magnitude higher than what is being observed from luminosity measurements and galaxy counts. The mean mass density of the Universe is usually obtained from the standard relations

$$\rho = \frac{M_u}{\frac{4}{3}\pi R_u^3} = \frac{qH^2}{\frac{4}{3}\pi G}, \qquad (m/l^3) \qquad (125)$$

where G is the free space value of the gravitational constant and H is Hubble's constant. A typical value of $H = 50\ \text{km s}^{-1}\ \text{Mpc}^{-1}$ will yield a cosmic mass density of $1\times10^{-26}\ \text{kg m}^{-3}$ that is much too high compared to the observed $\rho \approx 2\times10^{-30}\ \text{kg m}^{-3}$. The dimensionless constant q is the so called "acceleration parameter" and must equal unity or the Virial Theorem is violated. However, an acceleration parameter considerably less than unity at maximum recession velocities is often suggested as a solution to the mass density problem but proof of its validity is lacking. The "Einstein - de Sitter" model of the Universe, for example, requires that $q = \frac{1}{2}$, which is still too high to account for the above discrepancy in mass density. If Hubble's constant is defined as $H = v/R$ or $H = c/R_u$ where c is the limiting speed of recession and R_u the radius of curvature of the Universe, then Equation (125) is simply a variation of the Virial Theorem and can rewritten as

$$\frac{GM_u}{R_u^2} = \frac{c^2}{R_u} = a_0 \quad \text{(acceleration)}\,, \qquad (l/t^2) \quad (126)$$

or

$$\frac{GM_u}{R_u} = c^2 \quad \text{(velocity }^2\text{)}. \qquad (l^2/t^2) \quad (127)$$

The exact interpretation of Hubble's constant is, therefore, angular velocity and its physical dimensions $H = v/R$ inform us that we are part of a harmonic motion rather than a linearly expanding Universe.

If the Virial Theorem is truly universal, it must be valid not only for subatomic particles and planets, but also for galaxies and for the entire Universe. Consequently, it should be possible to construct a cosmological model, based on the Virial Theorem, and assess the result against known physical parameters. A simple approach would be to consider the gravitational interaction between the smallest known quantum of matter, the electron, and the rest of the Universe. In a virial Universe (where $M_u \propto R_u^2$) the ratio of the gravitational energy of the whole Universe to the gravitational energy of the smallest known matter (the electron) equals

$$\frac{GM_u^2/R_u}{Gm_e^2/r} = \left(\frac{R_u}{r}\right)^3 = \left(\frac{M_u}{m_e}\right)^{\frac{3}{2}} = \frac{R_u}{r_e}\frac{M_u}{m_e}\,, \qquad (none) \quad (128)$$

and the energy ratio of the electron’s attraction to the rest of the Universe and to its gravitational self energy is

$$\frac{GM_u m_e/R_u}{Gm_e^2/r_e} = \frac{R_u}{r_e} = \left(\frac{M_u}{m_e}\right)^{\frac{1}{2}} = \sqrt{N}\,. \qquad (none) \quad (129)$$

Equations (128) and (129) are easily modified to the Virial theorem

$$\frac{GM_u}{R_u^2} = \frac{Gm_e}{r_e^2} = a_0, \qquad (l/t^2) \quad (130)$$

where $a_0 = 7.62247 \times 10^{-12}\,\mathrm{m\,s^{-2}}$ is the rate of the cosmic acceleration or gravitational force per mass in the Universe which also equals the electron's gravitational surface acceleration. Since $GM_u / R_u^2 = c^2 / R_u$ it immediately follows that the radius of curvature of the Universe is

$$R_u = \frac{c^2}{a_0} = 1.17908 \times 10^{28}\,\mathrm{m}. \qquad (l) \quad (131)$$

The total mass within the radius R_u equals

$$M_u = \frac{R_u^2 a_0}{G} = 1.59486 \times 10^{55}\,\mathrm{kg}, \qquad (m) \quad (132)$$

and the mean mass density of the Universe becomes

$$\rho = \frac{M_u}{\frac{4}{3}\pi \quad R_u^3} = 2.32273 \times 10^{-30}\,\mathrm{kg\,m^{-3}}, \qquad (m/l^3) \quad (133)$$

which agrees with the observed value. The angular frequency of the expansion-contraction is given by

$$H = \omega_0 = \frac{c}{R_u} = 2.54258 \times 10^{-20}\,\mathrm{rad\,s^{-1}}, \qquad (t^{-1}) \quad (134)$$

which corresponds to an epoch, or period, of

$$t_0 = \frac{2\pi}{\omega_0} = 2.47118 \times 10^{20}\,\mathrm{s}. \qquad (t) \quad (135)$$

The potential energy of matter at R_u is

$$E = \frac{GM_u m}{R_u} = mc^2, \qquad (ml^2/t^2) \quad (136)$$

which conforms with Einstein's mass-energy relation $E = mc^2$.

The above equations again describe an oscillating (expanding-contracting) Universe which is about 100 times larger than current estimates and that is in perfect agreement with the large number hypotheses and the values listed in Table 1, Chapter 2, section 2.2. Whether we are part of a one or a multi-cycle Universe is determined by the rate of energy that is lost to heat or radiation. We have shown from the laws of harmonic motion, that a one cycle "deadbeat" (critically damped) Universe must dissipate all its potential energy over one period of oscillation, and the equivalent rate of radiation is

$$L_u = \frac{M_u c^2}{t_0} = 5.80044 \times 10^{51}\,\text{watts}. \qquad (ml^2/t^3) \quad (137)$$

Consequently, an electron must generate

$$\frac{\Delta E}{\Delta t} = \frac{m_e c^2}{t_0} = \frac{m_e a_0 c}{2\pi} = L_e, \qquad (ml^2/t^3) \quad (138)$$

or

$$L_e = \tfrac{1}{2}\ddot{h}, \qquad (ml^2/t^3) \quad (139)$$

which is the same as obtained by Equation (110).

The rate of radiation from Equations (137) matches the amount of radiation found in our Universe and the total amount of radiation from all matter determines the blackbody temperature of the Universe from Stefan's law:

$$T = \left(\frac{M_u c^2}{4\pi R_u^2 \sigma t_0} \right)^{\frac{1}{4}} = 2.766^\circ\,\text{K}, \qquad (none) \quad (140)$$

where $\sigma = 5.6705 \times 10^{-8}$ (Stefan-Boltzmann's constant). As can be seen, the Virial Theorem predicts exactly the same large number ratios as anticipated by Weyl, Stewart and Eddington.

A one cycle cosmological model based on the Virial Theorem does not comply with a constant value of Hubble's constant $H = \Delta v / \Delta R$. The problem is, that even if $H = \Delta v / \Delta R$ might appear linear on a small scale, it in fact follows the quadratic function v^2 / R which becomes more and more clear the further in space we probe. In contrast to other cosmological models, which usually contain vague numerology and therefore are difficult to challenge, the virial Universe provides exact solutions and can be experimentally tested. For example, from Equations (130), (138) and (139) it follows that

$$\frac{G}{\ddot{h}} = \frac{r_e^2 \pi}{m_e^2 c} \, , \qquad (lt / m^2) \qquad (141)$$

and if we replace the electron's dubious radius with the electromagnetic term $q^2 \mu_0 /(4\pi m_e)$ we can write

$$\frac{G}{\ddot{h}} = \frac{(q/m)^4 \mu_0^2}{16\pi c}. \qquad (lt / m^2) \qquad (142)$$

Since Planck's constant $\ddot{h}$ is accurately determined from experiments we again derive $G = 6.6445 \times 10^{-11}$ as the postulated free space value of the universal gravitational constant.

8.5 Conclusion

The Large Number Hypotheses and the Virial Theorem offer the same exact mathematical solutions as the harmonic Universe described previously. The idea that Ampère's law (see section 8.3) could provide a clue to gravitation is not new. Maxwell spoke of Ampère's discovery as "one of the most brilliant achievements in science. It is perfect in its form and unassailable in accuracy; and it is summed up in a formula from which all phenomena may be deduced" (Whittaker (1951)). It also seems proper to conclude with Eddington's own words: "A large ratio appears when we compare the electric force between a proton and electron with the gravitational force between them. I have long thought that this must be related to the number of particles in the Universe and I expect that the same view has been entertained by others. The above ratio is of the order of N."

The large number ratios seem to hint at a relationship where the mass of the Universe increases proportionally to the square of its radius or $M_u \propto R_u^2$ which might favor a non-uniform Universe, unless pancake shaped, as suggested by several investigators. It is difficult at the present time, to detect any non-uniformity in mass density over the entire Universe since the range of our most powerful telescopes at the present time is less than 1% of its radius R_u.