Einstein's Special Relativity Theory and Mach's Principle

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Abstract: Many historical works on Einstein describe his approval of Mach's philosophy and his effort to incorporate Mach's Principle into his relativity theories. Einstein eventually abandoned Mach's Principle but with some reservations. However, Mach's Principle still persists and its presumed incompatibility with Einstein's Relativity continues to be an obstacle for many in their attempts to understand Einstein's theory. This essay intends to resolve the issue by showing that Einstein's Special Relativity, in fact, is subject to Mach's Principle and that the proof can be found in the relativistic velocities of atomic orbits.

Ever since Einstein published his papers on Special Relativity [1,2] there have been many scientists who have not been fully satisfied with the theory. Perhaps most noteworthy is Walter Ritz, who collaborated with Einstein in 1909, and more recently the late Professor Petr Beckmann, who was the founder of the journal *Galilean Electrodynamics,* the principal aim of which is to refute Einstein's Special Theory of Relativity. From time to time other scientific journals have accepted articles critical of Special Relativity, and associations have been formed by philosophically minded groups who do not wholly accept the concept of space-time and the rejection of absolute space and absolute velocity, as upheld by the Special Theory of Relativity. One such organization is the Natural Philosophy Alliance, which boasts an impressive list of members.

However, scientists and philosophers who believe that Einstein's Special Theory of Relativity is one of the greatest achievements in science and irrefutable, by far outnumber those who are not convinced of its validity. Further, the relativistic velocity equations have been proven repeatedly in high energy particle accelerators.

I believe that for many scientists Special Relativity is difficult to understand except with respect to solving the equations. There is no doubt that Einstein's energy-velocity equation is valid and indisputable, and one of the greatest achievements in science. Why therefore, is there still scope for debate? It is the elimination of absolute space and absolute velocity or, in other words, the rejection of Mach's Principle [3,4], that

Einstein himself was forced to discard, which creates the conflict. It was in fact Einstein's mathematics teacher, Herman Minkowski [5], who introduced the purely mathematical concept of space-time, which discarded absolute space and absolute velocity, which Einstein reluctantly accepted [6].

It is possible to verify that Einstein was correct in believing that Mach's Principle should be incorporated into his relativity theory. Mach's Principle requires that inertia of mass and consequently potential energy of inertial mass must be generated by the rest of the Universe. In mathematical terms Mach's Principle can be written as $\phi_{univ} = GM_{univ} / R = c^2$ where ϕ_{univ} is the cosmic gravitational tension or the amount of potential energy per mass generated by the Universe; *G* is the gravitational constant; *c* the speed of light; *R* the absolute distance to the center of mass of the system and M_{univ} the total mass of matter within the radius of curvature *R*. Technically, Mach's Principle can be applied to the Earth and the solar system by simply using $\phi_{sol} = GM_{sol} / r = v^2$ where *r* is the distance to the center of mass of the solar system and M_{sol} the mass of the solar system within *r*. The gravitational tension ϕ_{sol} at the orbital radii *r* of the different planets equals the square of their orbital velocities. Mach's Principle can be further extended to our galaxy or to clusters of galaxies and ultimately to the Universe as a whole, at which point $\phi_{univ} = c^2$.

This leads to a velocity effect peculiar to Mach's Principle. For example, should we want to sling the Earth in its orbit at r_2 out to the orbit of Mars at r_3, then the amount of kinetic energy that needs to be added to Earth is $\Delta E = \frac{1}{2} m(\phi_2 - \phi_3)$, where *m* is the Earth's mass and ϕ_2 and ϕ_3 the gravitational tension of the solar system at r_2 and r_3 respectively. The difference in orbital velocity is thus $\Delta v = \sqrt{\phi_2} - \sqrt{\phi_3}$. However, decreasing the Earth's orbit by the same amount of energy, $\nabla E = \Delta E$ to a smaller radius r_1 means a loss of potential energy ($\nabla E =$loss of energy) in the form of friction and radiation or $\nabla E = \frac{1}{2} m(\phi_1 - \phi_2)$, and the difference in orbital velocity becomes $\nabla v = \sqrt{\phi_1} - \sqrt{\phi_2}$. Note, that when $\Delta E = \nabla E$ then $\Delta v \neq \nabla v$, which is a consequential effect of Mach's Principle.

The Special Theory of Relativity has so far ignored the above effect, since it considers matter at relative rest (thus the term rest mass energy $E_0 = m_0 c^2$) and cannot deduct velocities from rest or zero velocity. It can only accurately be applied to velocities that are produced by an increase in rest mass energy or $E_0 + \Delta E$. Einstein's relativistic velocity equation can be written in a Mach's format as

$$\Delta v = \sqrt{\phi_{univ} - \phi_{univ}\left(\frac{E_0}{E_0 + \Delta E}\right)^2} \ . \tag{1}$$

In cases where energy is lost to radiation such as when electrons are captured in high speed atomic orbits, Einstein's relativistic equation becomes obsolete and must be replaced by a second equation that can be used in cases where loss of rest mass energy occurs, such as $E_0 - \nabla E$ or

$$\nabla v = \sqrt{\phi_{univ} - \phi_{univ}\left(\frac{E_0 - \nabla E}{E_0}\right)^2} \ . \tag{2}$$

This becomes evident if we apply both the above velocity equations to the inner orbits of atoms and compare the results to published measured values that currently appear in *The Handbook of Chemistry and Physics* (under Ionization energies or Ionization potentials of the Elements).

The circumference of the innermost atomic orbit as determined by Louis de Broglie's wave theory is

$$\frac{\frac{1}{2}h\nabla v}{\nabla E} = \frac{Zq^2}{4\varepsilon_0 \nabla E}, \qquad (\tfrac{1}{2}\text{ wavelength}) \tag{3}$$

and solving for ∇E by inserting ∇v from Equation (2) we obtain

$$\nabla E_e = E_0\left[1 - \sqrt{1 - \left(\frac{Zq^2}{2\varepsilon_0 hc}\right)^2}\,\right] \times \frac{m_n}{m_n + m_e}, \qquad \text{(Joules)} \tag{4}$$

where Z is the atomic number; E_0 the electron's rest mass energy; q the electron's electric charge; ε_0 the permittivity constant and h Max Planck's constant. The term $m_n/(m_n + m_e)$ where m_n and m_e are masses of the atomic nucleus and electron respectively, reduces the orbital energy to that of the electron only. For example, Z=29 (Cu) yields a $\nabla E_e / q = 11573.35\ eV$ or $5.7\ eV$ higher than the published data.

Inserting Δv for Z = 29 from Einstein's Special Relativity Equation (1) into the above de Broglie Equation (3)

$$\Delta E_e = E_0\left[\left(\frac{1}{\sqrt{1 - (Zq^2/(2\varepsilon_0 hc))^2}}\right) - 1\right] \times \frac{m_n}{m_n + m_e}, \quad \text{(Joules)} \tag{5}$$

results in a critical error of 274 eV higher than published data which has prompted investigators to introduce several correction factors such as the Dirac-Fock correction [7]; self energy correction [8]; Uehling vacuum polarization correction [9]; higher order vacuum polarization correction [10] and nuclear size correction *etc.,* in order to match the measured values.

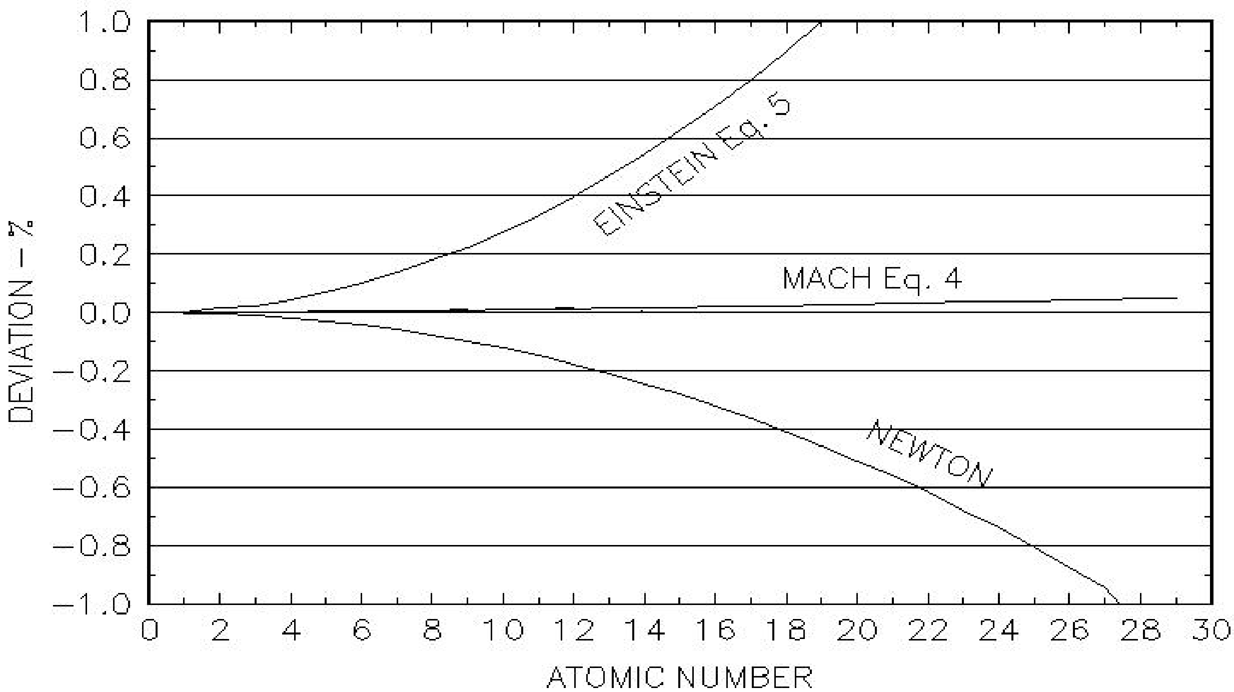

Fig. 1. Deviation in percent between measured values and results obtained from Equations (4) and (5). Also shown are values obtained from Newton's non-relativistic Equation.

The curves in Fig. 1, which are constructed from Equations (4) and (5) and Newton's non-relativistic relation $\Delta E = \frac{1}{2} m v^2$, show a deviation from measured values in percent. The results of Equation (5) seem to indicate a very small systematic Compton red shift of $\nabla \lambda = (1 - \cos \alpha) h / (m_e c)$ in the published measurements which accounts for the 5.7 eV discrepancy at Z=29. The Compton red shift could be caused by a recoil or deflection angle of $\cos \alpha = (k_1 \log eV) + k_2$ affecting the spectrometric measurements where $k_1 = 0.0197565$ and $k_2 = 0.89794$ are proportionality constants and the energy of the spectra in electron volts, see Fig. 2.

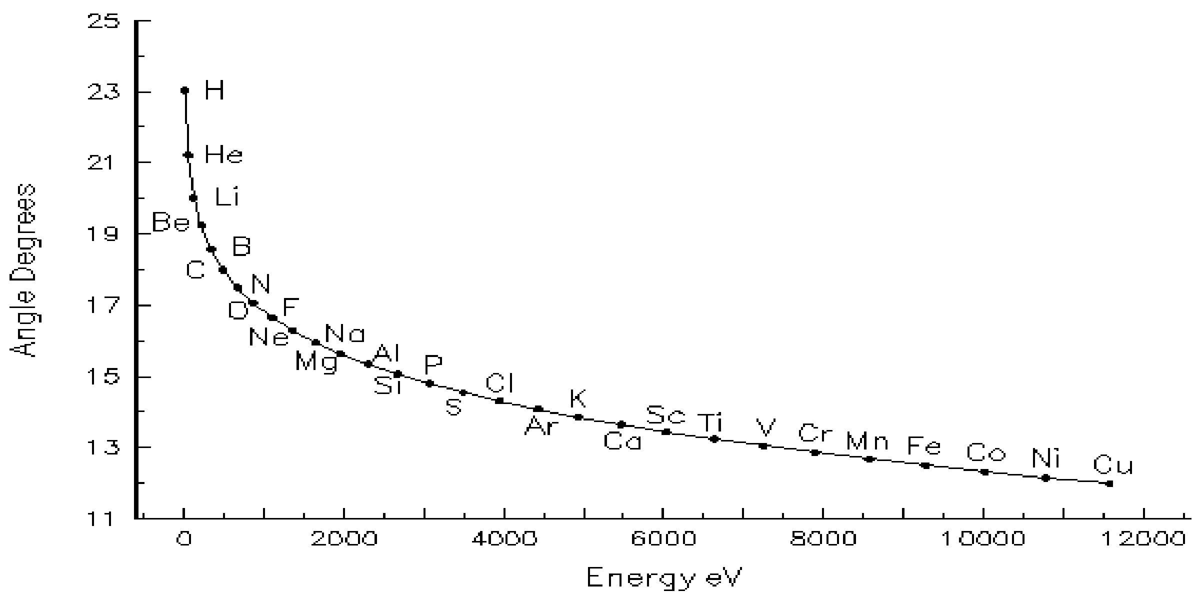

Fig. 2. Compton scattering angle α for the different elements

$$\alpha = \cos^{-1}\left[(0.0197565 \log eV_1) + 0.89794\right]$$

My personal conclusion is that the mathematics of Einstein's Special Theory of Relativity is only correct for cases of relative increase in rest mass energy, as in particle accelerators for example, and that Mach's Principle should be included in the theoretical interpretation of the theory to account for the energy-velocity relationship in cases where rest mass energy is lost, such as in atomic orbits. The close agreement between measured values and the results from Equation (4), after corrected for Compton red shifts, see Fig. 3, should prove this point.

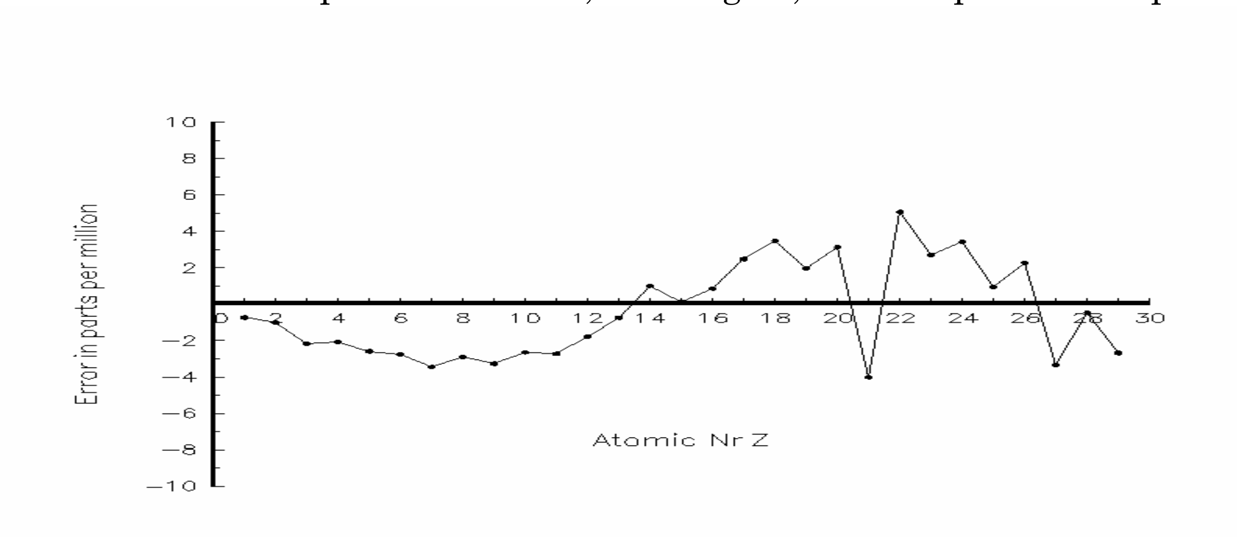

Fig. 3. Difference in parts per million between measured values (corrected for Compton red shifts) and the theoretical values from Equation (4).

The fact that Einstein's Special Theory of Relativity works well in particle accelerators, but fails for atomic orbits, is quite serious since there are by far more atoms in the world than particle accelerators.

It is remarkable that Mach's Principle has to be invoked in order to explain relativistic atomic orbits when Mach himself did not believe in atoms while Einstein, on the other hand, who was first to prove that atoms exist (Brownian movement and the photoelectric effect) chose to abandon Mach's Principle

References:

[1] A. Einstein, *AdP* **17,** 891, (1905).
[2] A. Einstein, *AdP* **20,** 627, (1906).
[3] E. Mach, *History and Root of the Conservation of Energy,* (1872).
[4] E. Mach, *The Science of Mechanics,* 6th ed., (1904).
[5] H. Minkowski, *Goett. Nachr.,* 53, (1908).
[6] A. Pais, *Subtle is the Lord,* Oxford University Press, Oxford, (1982).
[7[J.P. Desclaux, *Numerical Dirac-Fock Calculations for Atoms: Relativistic effects in Atoms, Molecules and Solids,* ed. G.L. Malli, Plenum Press, New York, p.133,(1981).
[8] P.J. Mohr, *Proceedings of the Workshop on Foundations of the Relativistic Theory of Atomic Structure,* ANL-80-126, (1981).
[9] E.A. Uehling, *Polarization effects in Positron Theory,* Phys. Rev., **48,** 55. [1935].
[10] G. Källen *et al., Fourth Order Vacuum Polarization,* Kong. Dansk. Vidensk. Selsk. Mat. Fys. Med., **29,** p.17 (1972).

WORK SHEET

Measured groundstate energies (highest ionization potential) for one-electron atoms from *Handbook of Chemistry and Physics*

TABLE 1

Z	Element	Energy eV_1	Z	Element	Energy eV_1	Z	Element	Energy eV_1
1	H	13.59844	11	Na	1648.702	21	Sc	6033.712
2	He	54.41778	12	Mg	1962.665	22	Ti	6625.82
3	Li	122.45429	13	Al	2304.141	23	V	7246.12
4	Be	217.71865	14	Si	2673.182	24	Cr	7894.81
5	B	340.22580	15	P	3069.842	25	Mn	8571.94
6	C	489.99334	16	S	3494.1892	26	Fe	9277.69
7	N	667.046	17	Cl	3946.296	27	Co	10 012.12
8	O	871.4101	18	Ar	4426.2296	28	Ni	10775.40
9	F	1103.1176	19	K	4934.046	29	Cu	11567.617
10	Ne	1362.1995	20	Ca	5469.864			

Above values corrected for a minor Compton red-shift of $\nabla\lambda = (1-\cos\alpha)h/(m_e c)$, where $\cos\alpha = (0.0197565 \log eV_1) + 0.8979399$, and by using the following equation:

$$eV_2 = eV_1 + \frac{\nabla\lambda\, q\,(eV_1)^2}{ch}.$$

TABLE 2

Z	Element	Energy eV_2	Z	Element	Energy eV_2	Z	Element	Energy eV_2
1	H	13.59846	11	Na	1648.906	21	Sc	6035.661
2	He	54.41817	12	Mg	1962.943	22	Ti	6628.102
3	Li	122.4560	13	Al	2304.511	23	V	7248.77
4	Be	217.72383	14	Si	2673.662	24	Cr	7897.86
5	B	340.23758	15	P	3070.453	25	Mn	8575.44
6	C	490.01632	16	S	3494.955.	26	Fe	9281.678
7	N	667.0862	17	Cl	3947.241	27	Co	10016.63
8	O	871.4754	18	Ar	4427.3808	28	Ni	10780.48
9	F	1103.217	19	K	4935.432	29	Cu	11573.322
10	Ne	1362.3452	20	Ca	5471.515			

Theoretical values using Equation

$$eV_3 = \frac{E_0}{q}\left[1-\sqrt{1-\left(\frac{Zq^2}{2\varepsilon_0 hc}\right)^2}\right] \times \frac{m_n}{(m_n+m_e)}$$

$$m_n/(m_n+m_e) = 1821\text{A}/(1821\text{A}+1)$$

TABLE 3

Z		A	Energy eV_3	Z		A	Energy eV_3	Z		A	Energy eV_3
1	H	1.008	13.598470	11	Na	22.99	1648.9105	21	Sc	44.96	6035.6852
2	He	4.003	54.418224	12	Mg	24.32	1962.9465	22	Ti	47.9	6628.0684
3	Li	6.94	122.456266	13	Al	26.98	2304.5127	23	V	50.95	7248.7505
4	Be	9.013	217.72428	14	Si	28.09	2673.6593	24	Cr	52.01	7897.8330
5	B	10.82	340.23846	15	P	30.975	3070.4526	25	Mn	54.94	8575.4320
6	C	12.011	490.01767	16	S	32.66	3494.9520	26	Fe	55.85	9281.6569
7	N	14.008	667.08850	17	Cl	35.457	3947.2312	27	Co	58.94	10016.6635
8	O	16.0	871.47793	18	Ar	39.94	4427.3654	28	Ni	58.71	10780.485
9	F	19.0	1103.2206	19	K	39.1	4935.4223	29	Cu	63.54	11573.353
10	Ne	20.183	1362.3488	20	Ca	40.08	5471.4978				

h = 6.626075540 E-34, c = 2.99792458 E+8, ε_0 = 8.854187817 E-12, m_e = 9.109389754 E-31, Proton-electron mass ratio $m_p/m_e = 1836$, q=1.6021773349 E-19, $E_0 = m_e c^2$=8.18711121654 E-14